Neutrino mixing as a source of dark energy

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We show that the vacuum condensate due to neutrino mixing in quantum field theory (QFT) contributes to the dark energy budget of the universe which gives rise to the accelerated behavior of cosmic flow. The mechanism of neutrino mixing is therefore a possible candidate to contribute to the cosmological dark energy.

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Data coming from cosmic microwave background radiation (CMBR) [1, 2], large scale structure [3, 4] and type Ia supernovae [5], used as standard candles, independently support the picture that the today observed universe can be consistently described as an accelerating Hubble fluid where the contribution of dark energy component to the total matter-energy density is $\Omega_\Lambda \simeq 0.7$. The big challenge is then the one of explaining such a bulk of dark energy component.

On the other hand, in recent years great attention has been devoted to the neutrino mixing phenomenon. The theoretical investigation of the neutrino mixing, firstly proposed by Pontecorvo [6], has been pursed in depth [7, 8, 9, 10, 11, 12], and more recently the issue of the construction of the flavored space of states has been settled in the framework of the quantum field theory (QFT) formalism [13, 14, 15, 16, 17, 18, 19] with the discovery of the unitary inequivalence between the flavored vacuum and the massive neutrino vacuum [13, 14], the associated finding of the neutrino-antineutrino pair condensate contributing to the vacuum energy [20] and the new oscillation formulas [14, 15, 16, 17]. The recent experimental achievements proving neutrino oscillations [21, 22] and the progresses in the QFT theoretical understanding [19] of the neutrino mixing thus provide a challenging and promising path beyond the Standard Model of electroweak interaction for elementary particles.

In this paper we show that these two most interesting issues are intimately bound together in such a way that one of them, namely the neutrino mixing phenomenon, appears to provide a contribution, till now unsuspected, to the vacuum dark energy component. The structure of the flavor vacuum and its unitary inequivalence to the vacuum for the massive neutrinos play a significant rôle in obtaining the result we report below.

We want to stress that in this paper we do not solve the problem of the arbitrariness of the momentum cut-off. The cut-off problem should therefore not be confused with the main message we want to present in this paper, namely that the very same phenomenon of the neutrino mixing appears to provide a contribution to the cosmological dark energy component. As shown below, such a feature, unsuspected till now, clearly manifests itself provided the mathematically correct QFT treatment of neutrino mixing [13, 14, 15, 16, 17, 18, 19, 20] is considered.

In the simplest explanation, the so called ΛCDM model, the cosmological constant contributes for almost 70% to the total matter-energy density budget. The standard theory of cosmological constant is based on the fact that the vacuum zero point energy cannot violate the Lorentz invariance of the vacuum and therefore the corresponding energy-momentum tensor density has the form $\mathcal{T}^{vac}_{\mu\nu} = \langle 0|\mathcal{T}_{\mu\nu}|0\rangle = \rho_\Lambda g_{\mu\nu}$, where ρ_Λ is a constant, i.e. a Lorentz scalar quantity. In the traditional picture the vacuum itself can be thought as a perfect fluid, source of the Einstein field equations and one derives [23] the equation of state $p_\Lambda = w\rho_\Lambda$, with p_Λ denoting the vacuum pressure and the adiabatic index w equals -1. As well known [24], one of the central pillars of Lorentz invariant local QFT is the very same definition of the vacuum state according to which it is the zero eigenvalue eigenstate of the normal ordered energy, momentum and angular momentum operators. Therefore, excluding by normal ordering zero-point contributions, any non-vanishing vacuum expectation value of one of these operators signals the breakdown of Lorentz invariance, since the vacuum would be dependent on space and/or time. The Lorentz invariance vacuum therefore implies $\mathcal{T}^{vac}_{\mu\nu} = \langle 0| : \mathcal{T}_{\mu\nu} : |0\rangle = 0$, (as usual normal ordering is denoted by the colon : ... :).

Usually, in the jargon one roughly expresses the Lorentz invariant characterization of the QFT vacuum, by saying that preserving the Lorentz invariance requires to exclude that kinematical terms in the energy-momentum tensor may contribute to the vacuum expectation values.

Suppose that, as said above, the contribution of the (zero point) vacuum energy density is taken to be equivalent to that of the cosmological constant Λ, which is expressed by $\rho_\Lambda = \Lambda/(8\pi G)$. Then, however, it turns out that the vacuum expectation value of the energy-momentum tensor is divergent, both for bosonic and fermionic fields, and this shortcoming can be addressed as the *cosmological constant problem*. By choosing to regularize the energy-momentum tensor by an ultraviolet cut-off at Planck scale, one gets a huge value for the vacuum energy density $\rho_{vac} \simeq c^5/G^2\hbar \sim 10^{76} GeV^4$ which is 123

orders of magnitude larger than the currently observed $\rho_\Lambda \simeq 10^{-47} GeV^4$ [25]. Also using a quantum chromodynamics (QCD) cut-off [26] the problem is not solved since $\rho_\Lambda^{QCD} \sim 10^{-3} GeV^4$ is still enormous with respect to the actual observed value.

Furthermore, there is another aspect which has to be taken into account: observations point out that cosmic flow is "today" accelerating while it was not so at intermediate redshift z (e.g. $1 < z < 10$). This situation gave rise to structure formation during the matter dominated era [27, 28]. This is an indication of the fact that any realistic cosmological model should roughly undergo four phases: an early accelerated phase (inflation), intermediate decelerated phases (radiation and matter dominated) and a final, today observed, accelerated phase. Obviously, the dynamical evolution (time dependence) of the cosmological constant through the different phases, breaks the Lorentz invariance of the vacuum and one has to face the problem of the dynamics for the vacuum energy in order to match the observations. In this case, therefore, we are not properly dealing with cosmological constant. Rather, we have to take into account some form of *dark energy* which *evolves* from early epochs inducing the today observed acceleration. Such a dynamical evolution of the dark energy, namely time dependence of the energy vacuum expectation value, violates the Lorentz invariance.

In the literature there are many proposals to achieve cosmological models justifying such a dark energy component, ranging from quintessence [29], to braneworld [30], to extended theories of gravity [31]. These approaches essentially consist in adding new ingredients to the dynamics (e.g. scalar fields), or in modifying cosmological equations (e.g. introducing higher order curvature terms in the effective gravitational action).

In this letter we show that, due to the condensate of neutrino-antineutrino pairs, the vacuum expectation value of the energy-momentum tensor naturally provides a contribution to the dark energy ρ_{vac}^{mix}, which in the early universe satisfies the strong energy condition (SEC) $\rho_{vac}^{mix} + 3p_{vac}^{mix} \geq 0$, and at present epoch behaves approximatively as a cosmological constant. Here p_{vac}^{mix} is the vacuum pressure induced by the neutrino mixing. Under such a new perspective, the energy content of the vacuum condensate could be substantially interpreted as dynamically evolving dark energy.

The main features of the QFT formalism for the neutrino mixing are summarized as follows. For the sake of simplicity, we restrict ourselves to the two flavor case [13]. Extension to three flavors can be found in Ref. [17]. The relation between the Dirac flavored neutrino fields $\nu_e(x)$, $\nu_\mu(x)$ and the Dirac massive neutrino fields $\nu_1(x)$, $\nu_2(x)$ is given by

$$\begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix} \tag{1}$$

being θ the mixing angle. The mixing transformation (1) can be written as $\nu_\sigma(x) \equiv G_\theta^{-1}(t)\ \nu_i(x)\ G_\theta(t)$, where $(\sigma, i) = (e,1), (\mu,2)$, and $G_\theta(t)$ is the transformation generator. The flavor annihilation operators are defined as $\alpha^r_{\mathbf{k},\sigma}(t) \equiv G_\theta^{-1}(t)\ \alpha^r_{\mathbf{k},i}\ G_\theta(t)$ and $\beta^r_{-\mathbf{k},\sigma}(t) \equiv G_\theta^{-1}(t)\ \beta^r_{-\mathbf{k},i}\ G_\theta(t)$. They annihilate the flavor vacuum $|0(t)\rangle_{e,\mu} \equiv G_\theta^{-1}(t)\ |0\rangle_{1,2}$, where $|0\rangle_{1,2}$ is the vacuum annihilated by $\alpha^r_{\mathbf{k},i}$ and $\beta^r_{-\mathbf{k},i}$.

The crucial point of our discussion is that $|0(t)\rangle_{e,\mu}$, which is the physical vacuum where neutrino oscillations are experimentally observed, is [13] a (coherent) condensate of $\alpha_{\mathbf{k},i}$ ($\beta_{\mathbf{k},i}$) neutrinos (antineutrinos):

$${}_{e,\mu}\langle 0|\alpha^{r\dagger}_{\mathbf{k},i}\alpha^r_{\mathbf{k},i}|0\rangle_{e,\mu} = {}_{e,\mu}\langle 0|\beta^{r\dagger}_{\mathbf{k},i}\beta^r_{\mathbf{k},i}|0\rangle_{e,\mu} = \sin^2\theta\ |V_{\mathbf{k}}|^2, \tag{2}$$

where $i = 1,2$, the reference frame $\mathbf{k} = (0,0,|\mathbf{k}|)$ has been adopted for convenience, $V_{\mathbf{k}}$ is the Bogoliubov coefficient entering the mixing transformation (see for example Refs. [13, 17]) and $|0\rangle_{e,\mu}$ denotes $|0(t)\rangle_{e,\mu}$ at a conventionally chosen time $t = 0$. As a consequence of its condensate structure the physical vacuum $|0(t)\rangle_{e,\mu}$ turns out to be unitary inequivalent to $|0\rangle_{1,2}$ [13]. For brevity, we omit here to reproduce the explicit expression of $V_{\mathbf{k}}$ which can be found, e.g., in Refs. [13, 17, 19]. We only recall that $V_{\mathbf{k}}$ is zero for $m_1 = m_2$, it has a maximum at $|\mathbf{k}| = \sqrt{m_1 m_2}$ and, for $|\mathbf{k}| \gg \sqrt{m_1 m_2}$, it goes like $|V_{\mathbf{k}}|^2 \simeq (m_2 - m_1)^2/(4|\mathbf{k}|^2)$. For massless neutrinos, as well known, one does not have mixing ($m_1 = m_2 = 0$). The oscillation formulas for the flavor charges $Q_{e,\mu}(t)$ are obtained by computing their expectation values in the physical vacuum $|0\rangle_{e,\mu}$ [17, 19].

Let us now calculate the contribution ρ_{vac}^{mix} of the neutrino mixing to the vacuum energy density. We consider the Minkowski metric (therefore we use the notation $\eta^{\mu\nu}$ instead of $g^{\mu\nu}$). The particle mixing and oscillations in curved background will be analyzed in a separate paper. Eq. (2) suggests that the energy content of the physical vacuum gets contributions from the $\alpha_{\mathbf{k},i}$ and $\beta_{-\mathbf{k},i}$ neutrino condensate. Therefore, as customary in such circumstances, we compute the total energy $T_{00} = \int d^3x \mathcal{T}_{00}(x)$ for the fields ν_1 and ν_2,

$$: T^{00}_{(i)} := \sum_r \int d^3\mathbf{k}\, \omega_{k,i} \left(\alpha^{r\dagger}_{\mathbf{k},i}\alpha^r_{\mathbf{k},i} + \beta^{r\dagger}_{-\mathbf{k},i}\beta^r_{-\mathbf{k},i} \right), \tag{3}$$

with $i = 1,2$ and where $: ... :$ denotes the normal ordering of the $\alpha_{\mathbf{k},i}$ and $\beta_{-\mathbf{k},i}$ operators. Note that $T^{00}_{(i)}$ is time independent.

We remark that we have ${}_{e,\mu}\langle 0|\ :\ T^{00}_{(i)}\ :\ |0\rangle_{e,\mu} = {}_{e,\mu}\langle 0(t)|\ :\ T^{00}_{(i)}\ :\ |0(t)\rangle_{e,\mu}$, for any t, within the QFT formalism for neutrino mixing. The contribution ρ_{vac}^{mix} of the neutrino mixing to the vacuum energy density is thus obtained:

$$\frac{1}{V}\ {}_{e,\mu}\langle 0| \sum_i : T^{00}_{(i)} : |0\rangle_{e,\mu} = \rho_{vac}^{mix}\ \eta^{00}\ . \tag{4}$$

By using Eq.(2), we then have

$$\rho_{vac}^{mix} = \frac{2}{\pi} \sin^2\theta \int_0^K dk\, k^2 (\omega_{k,1} + \omega_{k,2}) |V_{\mathbf{k}}|^2, \tag{5}$$

where the choice of the cut-off K will be discussed below.

Similarly, the expectation value of $T^{jj}_{(i)}$ in the vacuum $|0\rangle_{e,\mu}$ gives the contribution p^{mix}_{vac} of the neutrino mixing to the vacuum pressure:

$$\frac{1}{V}\ {}_{e,\mu}\langle 0|\sum_i :T^{jj}_{(i)}: |0\rangle_{e,\mu} = p^{mix}_{vac}\ \eta^{jj}\ , \qquad (6)$$

where no summation on the index j is intended. Being, for each diagonal component,

$$:T^{jj}_{(i)}: = \sum_r \int d^3\mathbf{k}\, \frac{k^j k^j}{\omega_{k,i}} \left(\alpha^{r\dagger}_{\mathbf{k},i}\alpha^r_{\mathbf{k},i} + \beta^{r\dagger}_{-\mathbf{k},i}\beta^r_{-\mathbf{k},i}\right), \quad (7)$$

(no summation on repeated indices), in the case of the isotropy of the momenta: $k^1 = k^2 = k^3$, $3(k^j)^2 = k^2$, we have $T^{11} = T^{22} = T^{33}$ and the following equation holds

$$p^{mix}_{vac} \ = \ -\frac{2}{3\pi}\sin^2\theta \int_0^K dk k^4 \left[\frac{1}{\omega_{k,1}} + \frac{1}{\omega_{k,2}}\right] |V_{\mathbf{k}}|^2. \, (8)$$

Eqs.(5) and (8) show that Lorentz invariance is broken and $\rho^{mix}_{vac} \neq -p^{mix}_{vac}$ for any value of the masses m_1 and m_2 and independently of the choice of the cut-off. We observe that $w \simeq -1/3$ when the cut-off is chosen to be $K \gg m_1, m_2$, cf. Eqs.(5) and (8) and the discussion below for the choice of K.

It is worth stressing that the violation of the Lorentz invariance originates from the neutrino-antineutrino condensate structure of the vacuum. Indeed, as it appears from the computation reported above, in the absence of such a condensate, i.e. with $|V_{\mathbf{k}}|^2 = 0$, the vacuum expectation value of *each* of the $(0,0)$ and (j,j) components of the energy-momentum tensor would be zero. We also remark that the non-zero expectation value we obtain is time-independent since, for simplicity, we are considering the Minkowski metric. When the curved background metric is considered, $|V_{\mathbf{k}}|^2$ gets a dependence on time, as we will show in a forthcoming paper. In any case, the contribution to the vacuum expectation value of $T^{\mu\nu}$ is found to be non-vanishing, in the present computation (or in the curved background case), not because the adopted metric is flat (or not), but because of the non-trivial structure of physical vacuum due to the mixing phenomenon (which manifests itself in the non-vanishing of $|V_{\mathbf{k}}|^2$).

The above result holds in the early universe, when the universe curvature radius is comparable with the oscillation length. At the present epoch, in which the breaking of the Lorentz invariance is negligible, the non-vanishing vacuum energy density ρ^{mix}_{vac} compatible with Lorentz invariance cannot come from condensate contributions carrying a non-vanishing $\partial_\mu \sim k_\mu = (\omega_k, k_j)$, as it happens in Eqs.(3) and (7) (see also Eqs.(5) and (8)). This means that it can only be imputed to the lowest energy contribution of the vacuum condensate, approximatively equal to

$$\rho^{mix}_\Lambda = \sum_i m_i \int \frac{d^3x}{(2\pi)^3}\ {}_{e,\mu}\langle 0| :\bar{\nu}_i(x)\nu_i(x): |0\rangle_{e,\mu}. \quad (9)$$

Consistently with Lorentz invariance, the state equation is now $\rho^{mix}_\Lambda \sim -p^{mix}_\Lambda$, where explicitly

$$\rho^{mix}_\Lambda = \frac{2}{\pi}\sin^2\theta \int_0^K dk\, k^2 \left[\frac{m_1^2}{\omega_{k,1}} + \frac{m_2^2}{\omega_{k,2}}\right] |V_{\mathbf{k}}|^2. \quad (10)$$

The result (10) shows that, at present epoch, the vacuum condensate, coming from the neutrino mixing, can contribute to the dark energy component of the universe, with a behavior similar to that of the cosmological constant [20].

We observe that, since, at present epoch, the characteristic oscillation length of the neutrino is much smaller than the radius of curvature of the universe, the mixing treatment in the flat space-time, in such an epoch, is a good approximation of that in FRW space-time. More interesting is also the fact that, at present epoch, the space-time dependent condensate contributions, carrying a non-vanishing k_μ, are missing (they do not contribute to the energy-momentum vacuum expectation value). The modes associated to these missing contributions are not long-wave-length modes and therefore they are negligible in the present flat universe, i.e with respect to the scale implied by an infinite curvature radius.

Eqs. (5) and (10) show that the contribution to the dark energy induced from the neutrino mixing of course goes to zero in the no-mixing limit, i.e. when the mixing angle $\theta = 0$ and/or $m_1 = m_2$, and clearly also for massless neutrinos. However, those equations also show that the contribution depends on the specific QFT nature of the mixing: indeed, it is absent in the quantum mechanical (Pontecorvo) treatment, where $V_{\mathbf{k}}$ is anyhow zero. This confirms that the contribution discussed above is a genuine QFT non-perturbative feature and it is thus of different origin with respect to the ordinary vacuum energy contribution of massive spinor fields arising from a radiative correction at some perturbative order [32]. This leads us to believe that a neutrino–antineutrino asymmetry, if any, related with lepton number violation [33], would not affect much our result. We will consider the problem of such an asymmetry in a future work.

We call the reader attention on the fact that the discussion and the related results presented till now, which constitute the core of our message, are independent of the choice of the cut-off K. The cut-off problem is a distinct problem to which we do not have a solution at the present.

As shown in Ref. [33], in a dense background of neutrinos, as in the case of the early universe during the Big Bang Nucleosyntesis, flavor particle-antiparticle pairs are produced by mixing and oscillations with typical momentum $k \sim \frac{m_1+m_2}{2}$, the average mass of the neutrinos. In this connection, we note that in the literature [20, 34] it has been noticed that the observed order of magnitude $\rho^{mix}_\Lambda \sim 10^{-47} GeV^4$ can be reproduced by cutting the momentum range at the QFT neutrino scales $\sqrt{m_1 m_2}$ or $\frac{m_1+m_2}{2}$, with $\sin^2\theta \simeq 0.3$, m_i of order of $10^{-3} eV$, so that $\delta m^2 = m_2^2 - m_1^2 \simeq 8 \times 10^{-5} eV^2$. This reflects the presence of the $|V_{\mathbf{k}}|^2$ factor, with its momentum dependence,

in the above integrations, which points to the relevance of soft momentum (long-wave-length) modes.

In conclusion, we have shown that the vacuum condensate due to neutrino mixing contributes to the dark energy budget of the universe. Different behaviors of the vacuum expectation value of the energy-momentum tensor have been discussed referring to different boundary conditions in different universe epochs. Our result is independent of the choice of the cut-off.

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